Single Step optimal Control Of The Half Circular Legged Monopod

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Outline

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- Castigliano's Theorem
- Single Step Optimal Control Problem
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Introduction

- Legged robots have complex architecture because of their nonlinear dynamics and ground contact conditions.
- There are different types of robot platforms that have different leg numbers. One example of these robot platforms is hexapod RHex platform.
- One legged robots have been studied by many researchers because of the fact that single legged systems have simpler mechanical constructions to build and it is easy to analyze these systems because they can only exhibit only hopping gait.

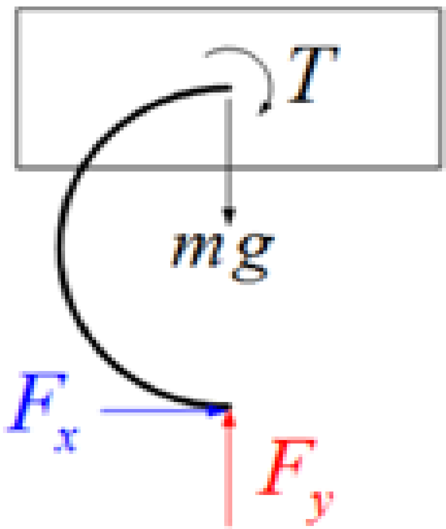
INTRODUCTION

- Raibert and his colleaques designed a hopping robot that is significant step for the legged robotics systems
- In this paper we used the single half circular legged robot. The robot has a body and a half circular compliant leg similar to RHex

Dynamic Equations of the Monopod

- We start with a single half circular legged model because it is simple to control and there is no problem with regulation of multiple legs.
- The robot has a rigid rectangular body with a mass and a compliant half circular leg.
- In dynamic equations we ignore the body pitching.
- The formulation of the equation of motion for the robot is based on the ground reaction forces that are calculated using Castigliano's Theorem.

Dynamic Equations of the Monopod



- **The Forces and Torque Acted on the Robot**

x : horizontal position of the COM

y: vertical position of the COM

θ : angular position of the leg wrt. ground

$$\ddot{x} = \frac{F_x}{m}$$

$$\ddot{y} = \frac{F_y - mg}{m}$$

$$\ddot{\theta} = \frac{T - F_x y - F_y (x_c - x)}{I_{leg}}$$

Dynamic Equations of the Monopod

- There is a relation between angular velocity and forward speed of the robot,

$$\dot{\theta} = \frac{2\dot{x}}{L} \quad \text{where} \quad L = R\sqrt{2(1+\cos\theta)}$$

- Since the leg exhibits rolling motion, the velocity of the ground contact point is equal to

$$\dot{x}_c = \frac{R\dot{\theta}}{2}$$

Dynamic Equations of the Monopod

- And also there is a kinematic relation between contact point x_c and position of the hip x

$$x_c = x + R\cos\theta \qquad \text{and}$$

$$\dot{x}_c = \dot{x} + R\cos\theta\dot{\theta} + \dot{R}\sin\theta$$

neglect $\dot{R}$

$$\dot{x} = \frac{-R\cos\theta\dot{\theta}}{1 - \dfrac{1}{2\sqrt{1+\cos\theta}}}$$

Dynamic Equations of the Monopod

- In the flight phase, the ground reaction forces act on the robot are equal to zero and the body exhibits projectile motion. There is only gravitational force that acts on the body.

$$\ddot{x} = 0$$

$$\ddot{y} = -g$$

$$\ddot{\theta} = \frac{T}{I_{leg}}$$

Castigliano's Theorem

- We used Castigliano's Theorem to model the deflection properties of the compliant half circular leg of the robot under the effect of external forces

CASTIGLIANO'S THEOREM

- According to Castigliano's Theorem,

the deflections at the point where the external forces are applied can be determined by the partial derivative of the total strain energy of the compliant leg with respect to that forces; i.e.,

the deflection amount on the direction of ith external force

$$\delta_i = \frac{\delta U}{\delta F_i}$$

the total strain energy

ith external force

Castigliano's Theorem

- The total strain energy of the half circular leg can be simplified to

$$U \approx \int \frac{M^2 R}{2EI} d\gamma$$

E : is the Young modulus
I : is the second moment of the inertia of the leg

which considers only moment of the external forces at the cross section, if the radius of the leg is ten times larger than its thickness

Castigliano's Theorem

- So the deflection of the loading point in the direction of external forces can be expressed as a linear function of the external forces, i.e,

$$\underbrace{\begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}}_{Deflections} = \underbrace{\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}}_{Compliant\ Matrix} \underbrace{\begin{bmatrix} F_x \\ F_y \end{bmatrix}}_{Forces}$$

- Note that, the elements of the compliant matrix are the function of the leg angle and radius of the leg

Single Step Optimal Control Problem

- We define two optimal control problems for each phase, stance and flight

- In the stance phase our aim is to maximize the final kinetic energy of the robot while keeping the leg radius near its rest length and we also try to find the initial conditions of the flight phase for a high jump.

- In the flight phase we find the optimal torque input that brings the leg to the desired final position.

Single Step Optimal Control Problem

- We try to obtain admissible control input that minimizes the performance measure

$$J(u) = h(z(t_f), t_f) + \int_{t_0}^{t_f} g(z(t), u(t), t)\,dt$$

where

$z(t) = [x, \dot{x}, y, \dot{y}, \theta, \dot{\theta}]$ is the state vector

$u(t)$ is the control input

Single Step Optimal Control Problem

- The Hamiltonian can be defined as a function of the state equations and performance measure as follows

$$H(z(t),u(t),p(t),t) = g(z(t),u(t),t) + p^T(t)a(z(t),u(t),t)$$

where $p(t)$ is the co-state vector. The derivative of the costate and state vectors are obtained by

SINGLE STEP OPTIMAL CONTROL PROBLEM

$$\dot{z}^{*} = \frac{\partial H}{\partial p}(z^{*}(t), u^{*}(t), p^{*}(t), t)$$

$$\dot{p}^{*} = \frac{\partial H}{\partial z}(z^{*}(t), u^{*}(t), p^{*}(t), t)$$

- Since the control input is unconstraint we used the following optimality condition:

$$0 = \frac{\partial H}{\partial u}(z^{*}(t), u^{*}(t), p^{*}(t), t)$$

Single Step Optimal Control Problem

- As stated before, during the stance phase we desire to keep the height of COM around the undeflected leg diameter and maximize the final total kinetic energy (*TKE*) given in

$$TKE = \frac{1}{2}m(x^2 + y^2) + \frac{1}{2}I_{leg}\theta^2$$

Single Step Optimal Control Problem

- According to desired conditions we can define the performance measure of stance phase

$$J_s(u) = h_s(z(t_f), t_f) + \int_{t_0}^{t_f} g_s(z(t), u(t), t)\,dt$$

where

$$h_s(z(t_f), t_f) = (y(t_f) - 2R_i)^2 - TKE(t_f)$$

$$g_s(z(t), u(t), t) = \frac{1}{2}(y - 2R_i)^2$$

Single Step Optimal Control Problem

- In the flight phase we attempt to obtain continuous rotation of the leg that ends with desired touchdown angle. The performance measure of the flight phase is

$$J_f(u) = h_f(z(t_f), t_f) + \int_{t_0}^{t_f} g_f(z(t), u(t), t)\,dt$$

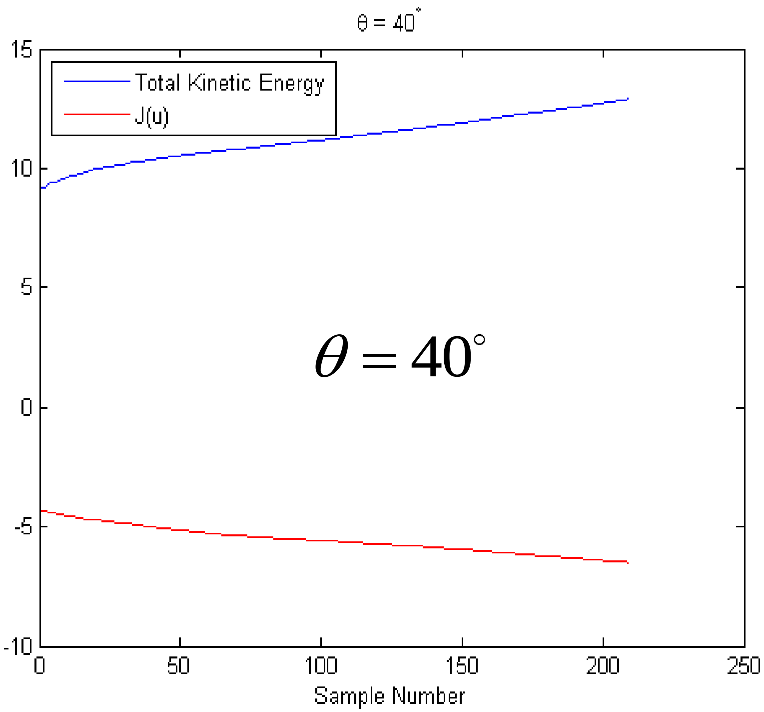
where

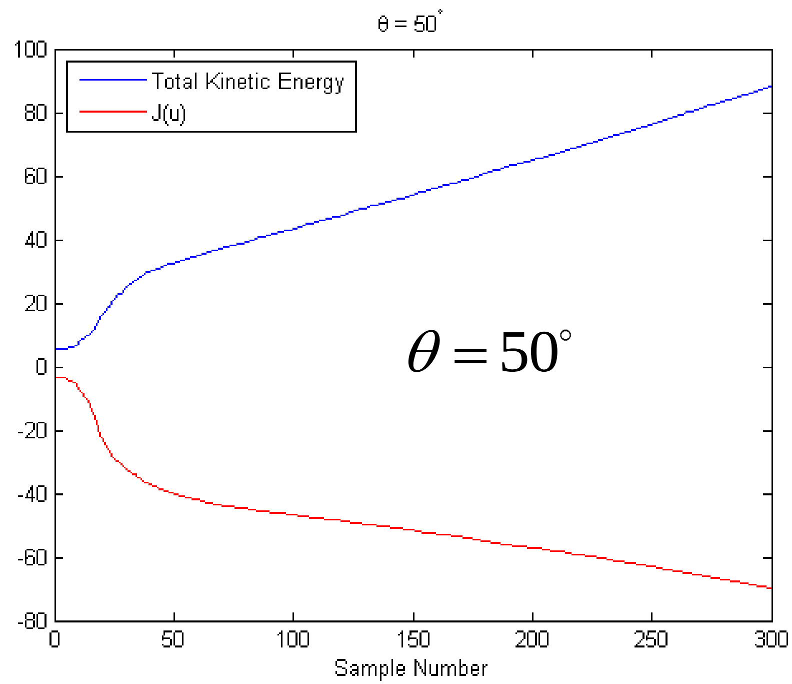
$$h_f(z(t_f), t_f) = (y(t_f) - R_i(\cos\theta(t_f) + 1))^2 + (\theta(t_f) - \theta_{des})^2$$

$$g_f(z(t), u(t), t) = \left(\dot{\theta} - \frac{\theta_{total}}{t_{flight}}\right)^2 \qquad t_{flight} = 2\left(\frac{\dot{y}}{g} + \sqrt{\frac{y_i - y_f}{g}}\right)$$

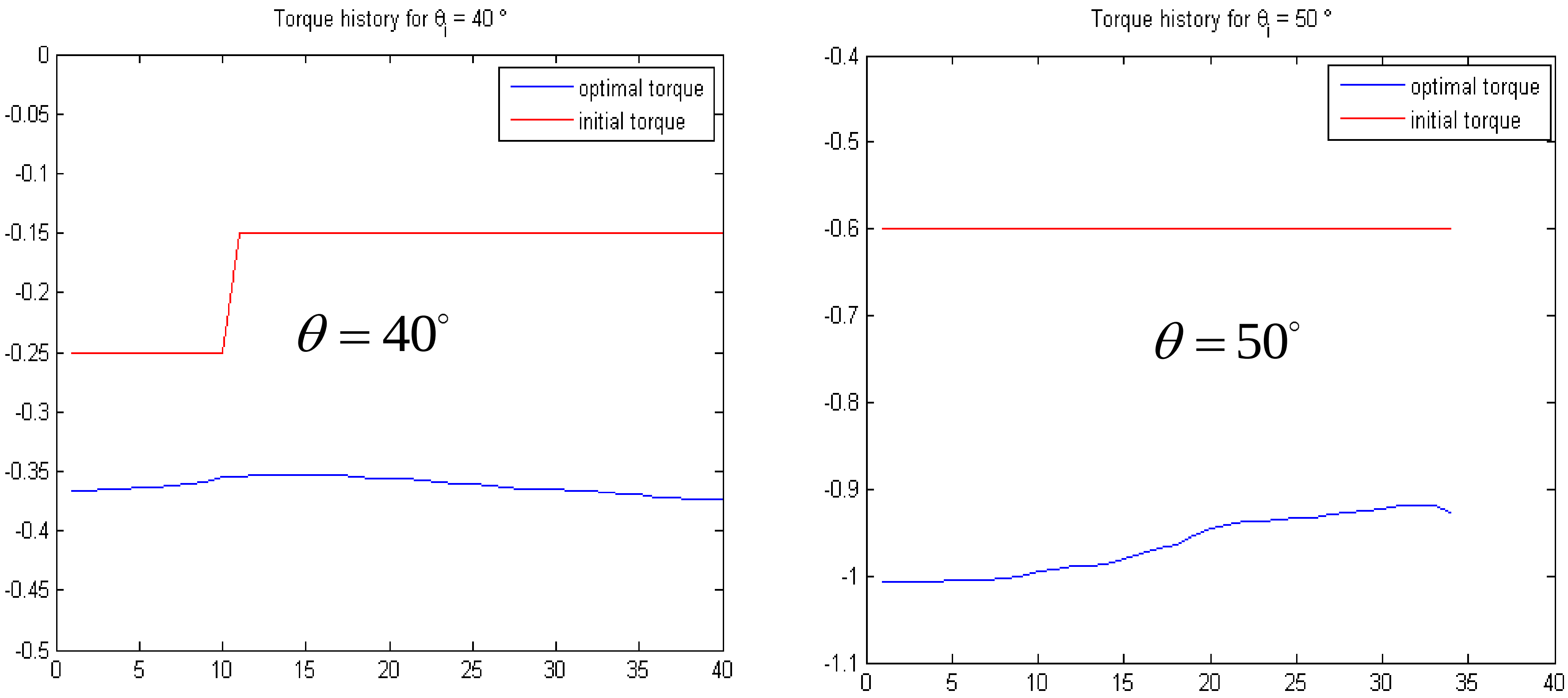
Numerical Results

- These two problems were solved by Steepest Descent Algorithm.





Iteration Number vs. TKE and J(u)



Initial and Optimal Torque Input

Conclusions

- We presented a controller that can be used for a single jump problem of monopod robot.

- The robot we used in this study has a single actuator located at hip point and rotates the half circular leg. The only input to the system is this actuator torque.

- The future work includes the extension of the single jump problem to the multiple jump problem.

QUESTIONS