A Type Synthesis Analysis For Generalized Stewart Platforms

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Content

Introduction

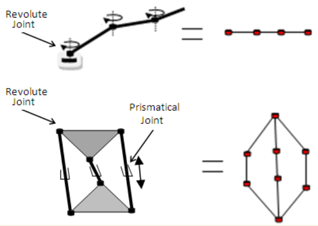
- Type synthesis is determining all possible types of Parallel Kinematic Machines (PKMs) which produce a specified motion pattern for the end effector.
- Gao et al. considered the type synthesis as a geometric constraint problem and introduced Generalized Stewart Platforms(GSPs).
- They found 3850 types of GSPs.
- We proposed two additional criteria to select more practicable structures among the 3850 types of GSPs
- After applying the criteria the number of possible types of GSPs reduced to 195.
- The all possible types of GSPs presented as tables.

Aim and motivation of the work

- The goal of this study is to make an analyse about type synthesis of PKMs and find more practical PKMs.
- There are many studies about type synthesis of the PKMs in the literature, however the study by Gao et al is quite different, since they define the problem as a geometrical constraint problem and produce GSPs . In the study they found the number of all possible types of GSPs but didn't give the all types of GSPs, the need for such a study makes of our primary motivation.
- Two additional criteria are defined to both reduce the possible number of GSPs and lead the resarches to find more practical structures.
- The results obtained offer a reference work to the researchers who consider to find *"new"* and more practical GSPs.

TYPE SYNTHESIS FOR PKMS (GRAPH THEORY BASED METHOD)

- Graph theory is one of the branches of the mathematics.
- Freudenstein proposed a graphical representation for mechanism's bodies and joints. In this representation vertexes in the graph indicates mechanism's bodies while lines indicates mechanism's joints.



TYPE SYNTHESIS FOR PKMS (GRAPH THEORY BASED METHOD)

- Two major problems are:
- Isomorphism: The problem about one to one mapping between graphs and mechanisms has not been completely solved yet. Therefore the same mechanism can be represent with several different graphs.
- Grubler Formula: Determination of mechanism DOF is defined used Grubler formula. However this formula is not valid for some kind of special mechanisms.

TYPE SYNTHESIS FOR PKMS (GROUP THEORY BASED METHOD)

- This method is based on a group that is related to the Special Euclidean SE(3) matrix group namely displacement group. This group represents motion of a rigid body.
- SE(3) matrix group is defined as

$$SE = \left\{ \begin{pmatrix} R & p \\ 0 & 1 \end{pmatrix} \right\}$$

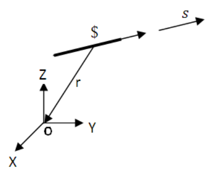
TYPE SYNTHESIS FOR PKMS (GROUP THEORY BASED METHOD)

- There are several subgroups of the displacement group which can be used for the type synthesis of PKMs. Some examples for these groups can be mentioned as;
- {T(u}: All translational motions parallel to u vector.
- {T}: All spatial translational motions.
- {X(w)}: All translational and rotational motions about all axes parallel to the axis of w vector.
- The different combinations of these subgroups are used for the type synthesis of PKMs.
- The main aim of these method is finding all of the group structures.

Type Synthesis for PKMs

(Screw Theory Based Method)

- Screw theory is firstly introduced by Ball in 1900.
- Chasles theorem states that any displacement of a rigid body in space can be represent by means of a rotation about an axis and then a translation parallel to that axis. The axis is named as screw axis while the ratio between translation and rotation is named as pitch.($d = h\theta$)



$$\$ = \begin{bmatrix} \$_F \\ \$_S \end{bmatrix} = \begin{cases} \begin{bmatrix} s \\ s \times r + hs \end{bmatrix}, & h \neq \infty \\ \begin{bmatrix} 0 \\ s \end{bmatrix}, & h = \infty \end{cases}$$

Type Synthesis for PKMs

(Screw Theory Based Method)

- Reciprocal Screws: Two screws are to be reciprocal if provide;

$$\$_1 \circ \$_2 = [\Pi \$_1]^T \$_2 = 0 \qquad \Pi = \begin{bmatrix} 0 & I_3 \\ I_3 & 0 \end{bmatrix}$$

- Twist and Wrench: A screw is called as a twist if it represents an instantaneous motion of a rigid body and a wrench if it represents a system of forces and couples acting on a rigid body.

Type Synthesis for PKMs

(Screw Theory Based Method)

- The main steps while type synthesis of PKMs using screw theory are :
- Determine the wrench system that is reciprocal to the desired twist system of the end-effector of the mechanism.
- Finding the kinematic chains' wrench system of the mechanism, their union spans the wrench system found in the first step.
- Determine all of the kinematic chains which are generate the corresponding wrenches.

Type Synthesis of 6 DOF GSPs as a Geometric Constraint Problem

- The study by Gao et al. (X. S. Gao, D. Lei, Q. Liao, and G. F. Zhang. Generalized Stewart-Gough platforms and their direct kinematics. IEEE Transactions on Robotics, pp. 21, 141-151, 2005)
- Three geometric primitives points, planes and lines in three-dimensional Euclidean space.
- Two types of geometric constrains angular and distance constraints.
- Six types of distance constraints:point-point, line-line, point-line, line-point, point-plane and plane-point.
- Four types of angular constraints: line-line, plane-plane, plane-line and line-plane.
- Four classes of GSPs: 3D3A GSPs, 4D2A GSPs, 5D1A GSPs and 6D GSPs.
- 3850 possible forms of GSPs with 6-DOF.

Type Synthesis of 6 DOF GSPs as a Geometric Constraint Problem

Constraint Name	Constraint Type	Symbol
D1	Point-Point Distance Constraint	DPP
D2	Line-Point Distance Constraint	DLP
D3	Point-Line Distance Constraint	DPL
D4	Line-Line Distance Constraint	DLL
D5	Point-Plane Distance Constraint	DPH
D6	Plane-Point Distance Constraint	DHP
A1	Line-Line Angular Constraint	ALL
A2	Line-Plane Angular Constraint	ALH
A3	Plane-Line Angular Constraint	AHL
A4	Plane-Plane Angular Constraint	AHH

Type Synthesis of 6 DOF GSPs as a Geometric Constraint Problem

- A GSP type can be display using its constraints in two ways.
- The first one is constraint base display: This type is performed by joining the constraints side by side.

$D_1D_1D_4D_4D_3A_1$

- The second display type is the symbol based type. In this type the symbols of the constraints get bring together as ordered pairs on both side of the "-" symbol according to their positions on the base and on the end-effector frames of the GSP.

PPLLPLa — PPLLLa

Geometric pirimitives On end-effector's frame — Geometric pirimitives On base frame

Type Synthesis of 6 DOF GSPs as a Geometric Constraint Problem (The Criteria)

- The first criterion disregards planar joints that are rarely preferred and restricts the motion in the plane only.
- The second criterion considers the symmetrical conditions given by Tsai
  - The number of kinematic chains in the GSP and the number of DOF of the end-effector of the GSP should be equal.
  - Each kinematic chain should be equal in terms of the number, type and order of the joints.
  - The type and order of the active joint in the each kinematic chain should be the same.

Type Synthesis of 6 DOF GSPs as a Geometric Constraint Problem (Computation of all Possible Types of GSPs)

- The number of GSPs in 3D3A class: The GSPs in this class have 3 distance constraints from four types of distance constraints ($D_1...D_4$) and 3 angular constraints from 1 type of angular constraint (A_1). 3 constraints from 4 type constraints can be select as follows (n=4 m=3);

$$C^m_{m+n-1} = C^3_{3+4-1} = C^3_6 = \frac{6.5.4.3!}{3!.3!} = 20 \qquad C^m_{m+n-1} = C^3_{3+1-1} = C^3_3 = 1$$

$D_1D_1D_1A_1A_1A_1$	$D_1D_2D_2A_1A_1A_1$	$D_1D_3D_4A_1A_1A_1$	$D_2D_2D_4A_1A_1A_1$
$D_1D_1D_2A_1A_1A_1$	$D_1D_2D_3A_1A_1A_1$	$D_1D_4D_4A_1A_1A_1$	$D_2D_3D_3A_1A_1A_1$
$D_1D_1D_3A_1A_1A_1$	$D_1D_2D_4A_1A_1A_1$	$D_2D_2D_2A_1A_1A_1$	$D_2D_3D_4A_1A_1A_1$
$D_1D_1D_4A_1A_1A_1$	$D_1D_3D_3A_1A_1A_1$	$D_2D_2D_3A_1A_1A_1$	$D_2D_4D_4A_1A_1A_1$
$D_3D_3D_3A_1A_1A_1$	$D_3D_3D_4A_1A_1A_1$	$D_3D_4D_4A_1A_1A_1$	$D_4D_4D_4A_1A_1A_1$

All possible types of 3D3A GSPs (20)

TYPE SYNTHESIS OF 6 DOF GSPS AS A GEOMETRIC CONSTRAINT PROBLEM

(COMPUTATION OF ALL POSSIBLE TYPES OF GSPS)

$D_1D_1D_1D_1A_1A_1$	$D_1D_1D_3D_3A_1A_1$	$D_1D_2D_3D_4A_1A_1$	$D_2D_2D_2D_3A_1A_1$
$D_1D_1D_1D_2A_1A_1$	$D_1D_1D_3D_4\ A_1A_1$	$D_1D_2D_4D_4A_1A_1$	$D_2D_2D_2D_4A_1A_1$
$D_1D_1D_1D_3A_1A_1$	$D_1D_1D_4D_4\ A_1A_1$	$D_1D_3D_3D_3A_1A_1$	$D_2D_2D_3D_3A_1A_1$
$D_1D_1D_1D_4A_1A_1$	$D_1D_2D_2D_2\ A_1A_1$	$D_1D_3D_3D_4A_1A_1$	$D_2D_2D_3D_4A_1A_1$
$D_1D_1D_2D_2A_1A_1$	$D_1D_2D_2D_3\ A_1A_1$	$D_1D_3D_4D_4A_1A_1$	$D_2D_2D_4D_4A_1A_1$
$D_1D_1D_2D_3A_1A_1$	$D_1D_2D_2D_4\ A_1A_1$	$D_1D_4D_4D_4A_1A_1$	$D_2D_3D_3D_3A_1A_1$
$D_1D_1D_2D_4A_1A_1$	$D_1D_2D_3D_3\ A_1A_1$	$D_2D_2D_2D_2A_1A_1$	$D_2D_3D_3D_4A_1A_1$
$D_2D_3D_4D_4A_1A_1$	$D_2D_4D_4D_4\ A_1A_1$	$D_3D_3D_3D_3A_1A_1$	$D_3D_3D_3D_4A_1A_1$
$D_3D_3D_4D_4A_1A_1$	$D_3D_4D_4D_4\ A_1A_1$	$D_4D_4D_4D_4A_1A_1$	**********

All possible types of 4D2A GSPs (35)

TYPE SYNTHESIS OF 6 DOF GSPS AS A GEOMETRIC CONSTRAINT PROBLEM

(COMPUTATION OF ALL POSSIBLE TYPES OF GSPS)

$D_1D_1D_1D_1D_1A_1$	$D_1D_1D_1D_3D_4A_1$	$D_1D_1D_3D_3D_3A_1$	$D_1D_2D_2D_3D_4A_1$
$D_1D_1D_1D_1D_2A_1$	$D_1D_1D_1D_4D_4A_1$	$D_1D_1D_3D_3D_4A_1$	$D_1D_2D_2D_4D_4A_1$
$D_1D_1D_1D_1D_3A_1$	$D_1D_1D_2D_2D_2A_1$	$D_1D_1D_3D_4D_4A_1$	$D_1D_2D_3D_3D_3A_1$
$D_1D_1D_1D_1D_4A_1$	$D_1D_1D_2D_2D_3A_1$	$D_1D_1D_4D_4D_4A_1$	$D_1D_2D_3D_3D_4A_1$
$D_1D_1D_1D_2D_2A_1$	$D_1D_1D_2D_2D_4A_1$	$D_1D_2D_2D_2D_2A_1$	$D_1D_2D_3D_4D_4A_1$
$D_1D_1D_1D_2D_3A_1$	$D_1D_1D_2D_3D_3A_1$	$D_1D_2D_2D_2D_3A_1$	$D_1D_2D_4D_4D_4A_1$
$D_1D_1D_1D_2D_4A_1$	$D_1D_1D_2D_3D_4A_1$	$D_1D_2D_2D_2D_4A_1$	$D_1D_3D_3D_3D_3A_1$
$D_1D_1D_1D_3D_3A_1$	$D_1D_1D_2D_4D_4A_1$	$D_1D_2D_2D_3D_3A_1$	$D_1D_3D_3D_3D_4A_1$
$D_2D_3D_4D_4D_4A_1$	$D_2D_4D_4D_4D_4A_1$	$D_3D_3D_3D_3D_3A_1$	$D_3D_3D_3D_3D_4A_1$
$D_3D_3D_3D_4D_4A_1$	$D_3D_3D_4D_4D_4A_1$	$D_3D_4D_4D_4D_4A_1$	$D_4D_4D_4D_4D_4A_1$
$D_2D_2D_2D_4D_4A_1$	$D_2D_2D_3D_3D_3A_1$	$D_2D_2D_3D_3D_4A_1$	$D_2D_2D_3D_4D_4A_1$
$D_2D_2D_4D_4D_4A_1$	$D_2D_3D_3D_3D_3A_1$	$D_2D_3D_3D_3D_4A_1$	$D_2D_3D_3D_4D_4A_1$
$D_1D_3D_3D_4D_4A_1$	$D_1D_3D_4D_4D_4A_1$	$D_1D_4D_4D_4D_4A_1$	$D_2D_2D_2D_2D_2A_1$
$D_2D_2D_2D_2D_3A_1$	$D_2D_2D_2D_2D_4A_1$	$D_2D_2D_2D_3D_3A_1$	$D_2D_2D_2D_3D_4A_1$

All possible types of 5D1A GSPs (56)

TYPE SYNTHESIS OF 6 DOF GSPS AS A GEOMETRIC CONSTRAINT PROBLEM

(COMPUTATION OF ALL POSSIBLE TYPES OF GSPS)

$D_1D_1D_1D_1D_1D_1$	$D_1D_1D_1D_2D_2D_3$	$D_1D_1D_2D_2D_2D_4$	$D_1D_1D_3D_4D_4D_4$	$D_1D_2D_2D_4D_4D_4$	$D_1D_4D_4D_4D_4D_4$	$D_2D_2D_3D_3D_3D_3$	$D_3D_3D_3D_3D_3D_3$
$D_1D_1D_1D_1D_1D_2$	$D_1D_1D_1D_2D_2D_4$	$D_1D_1D_2D_2D_3D_3$	$D_1D_1D_4D_4D_4D_4$	$D_1D_2D_3D_3D_3D_3$	$D_2D_2D_2D_2D_2D_2$	$D_2D_2D_3D_3D_3D_4$	$D_3D_3D_3D_3D_3D_4$
$D_1D_1D_1D_1D_1D_3$	$D_1D_1D_1D_2D_3D_3$	$D_1D_1D_2D_2D_3D_4$	$D_1D_2D_2D_2D_2D_2$	$D_1D_2D_3D_3D_3D_4$	$D_2D_2D_2D_2D_2D_3$	$D_2D_2D_3D_3D_4D_4$	$D_3D_3D_3D_3D_4D_4$
$D_1D_1D_1D_1D_1D_4$	$D_1D_1D_1D_2D_3D_4$	$D_1D_1D_2D_2D_4D_4$	$D_1D_2D_2D_2D_2D_3$	$D_1D_2D_3D_3D_4D_4$	$D_2D_2D_2D_2D_2D_4$	$D_2D_2D_3D_4D_4D_4$	$D_3D_3D_3D_4D_4D_4$
$D_1D_1D_1D_1D_2D_2$	$D_1D_1D_1D_2D_4D_4$	$D_1D_1D_2D_3D_3D_3$	$D_1D_2D_2D_2D_2D_4$	$D_1D_2D_3D_4D_4D_4$	$D_2D_2D_2D_2D_3D_3$	$D_2D_2D_4D_4D_4D_4$	$D_3D_3D_4D_4D_4D_4$
$D_1D_1D_1D_1D_2D_3$	$D_1D_1D_1D_3D_3D_3$	$D_1D_1D_2D_3D_3D_4$	$D_1D_2D_2D_2D_3D_3$	$D_1D_2D_4D_4D_4D_4$	$D_2D_2D_2D_2D_3D_4$	$D_2D_3D_3D_3D_3D_3$	$D_3D_4D_4D_4D_4D_4$
$D_1D_1D_1D_1D_2D_4$	$D_1D_1D_1D_3D_3D_4$	$D_1D_1D_2D_3D_4D_4$	$D_1D_2D_2D_2D_3D_4$	$D_1D_3D_3D_3D_3D_3$	$D_2D_2D_2D_2D_4D_4$	$D_2D_3D_3D_3D_3D_4$	$D_4D_4D_4D_4D_4D_4$
$D_1D_1D_1D_1D_3D_3$	$D_1D_1D_1D_3D_4D_4$	$D_1D_1D_2D_4D_4D_4$	$D_1D_2D_2D_2D_4D_4$	$D_1D_3D_3D_3D_3D_4$	$D_2D_2D_2D_3D_3D_3$	$D_2D_3D_3D_3D_4D_4$	$D_1D_3D_4D_4D_4D_4$
$D_1D_1D_1D_1D_3D_4$	$D_1D_1D_1D_4D_4D_4$	$D_1D_1D_3D_3D_3D_3$	$D_1D_2D_2D_3D_3D_3$	$D_1D_3D_3D_3D_4D_4$	$D_2D_2D_2D_3D_3D_4$	$D_2D_3D_3D_4D_4D_4$	$D_2D_2D_2D_4D_4D_4$
$D_1D_1D_1D_1D_4D_4$	$D_1D_1D_2D_2D_2D_2$	$D_1D_1D_3D_3D_3D_4$	$D_1D_2D_2D_3D_3D_4$	$D_1D_3D_3D_4D_4D_4$	$D_2D_2D_2D_3D_4D_4$	$D_2D_3D_4D_4D_4D_4$	$D_2D_4D_4D_4D_4D_4$
$D_1D_1D_1D_2D_2D_2$	$D_1D_1D_2D_2D_2D_3$	$D_1D_1D_3D_3D_4D_4$	$D_1D_2D_2D_3D_4D_4$	************	************	************	************

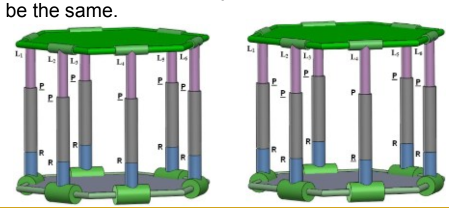
All possible types of 6D GSPs (84)

Total possiple types of GSPs=20+35+56+84=195

TYPE SYNTHESIS OF 6 DOF GSPS AS A GEOMETRIC CONSTRAINT PROBLEM

(SYMMETRICAL CONDITIONS)

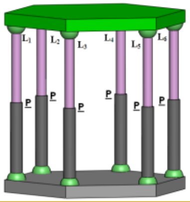
- The number of kinematic chains in the GSP and the number of DOF of the end-effector of the GSP should be equal.
- Each kinematic chain should be equal in terms of the number, type and order of the joints.
- The type and order of the active joint in the each kinematic chain should be the same.



TYPE SYNTHESIS OF 6 DOF GSPS AS A GEOMETRIC CONSTRAINT PROBLEM

(SYMMETRICAL CONDITIONS)

- The classes which contain angular constraints (3D3A, 4D2A and 5D1A) have not contain any symmetrical GSPs.
- The all GSPs in the 6D class are also not symmetrical since the GSPs which have different types of distance constraints cannot suit the second condition.
- A symmetrical GSP can be obtained if only the all of the it's constraints are selected from the just one type distance constraint.



Conclusion and Future Work

- In the present study an analyze about type synthesis of 6 DOF GSPs is presented.
- The study by Gao et al. is regarded as the base study.
- Two additional criteria are defined.
- The creteria are applied to the results of the study by Gao et al.
- The obtained 195 possible structures presented as tables.
- Future work includes kinematical and dynamical analysis of these structures.

Thanks

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