

# MODELING AND CONTROL OF A RIGID-LINK FLEXIBLE JOINT ROBOT MANIPULATOR

Ismail H. AKYUZ, Ali YUKSEL, Zafer BINGUL

*Department of Mechatronics Engineering,  
Kocaeli University, Kocaeli, TURKEY*

## INDEX

- *INTRODUCTION*
- *MATHEMATICAL MODELING*
- *SYSTEM DESIGN*
- *CONTROLLER DESIGN*
- *EXPERIMENTAL RESULTS*
- *CONCLUSION*

---

REM- 2011

1

## INTRODUCTION

### Where the flexible mechanisms are used?

- Servicing sector
- Various space station building and maintenance
- Gantry cranes
- Atomic force microscopes
- Medical and Defense industries.
- etc.




---

REM- 2011

2

## INTRODUCTION

### Why the flexible manipulators is needed?

- Increased payload capacity (greater the ratio of payload weight to robot weight)
- Reduced energy consumption (use of less powerful actuators)
- Cheaper construction (fewer materials and smaller actuators)
- Faster movements (higher accelerations because of lighter links)
- Longer reach (More access and space because of a more slender construction)
- Safer operation (no damage because of the compliance and low inertia)

---

REM- 2011

3

## INTRODUCTION

### What is the disadvantages of flexibility?

- Decreases the end-point accuracy
- Increases settling time
- Make the controller design scheme complicated

REM-2011

4

## INTRODUCTION

### There exist two different control approaches:

- Linear Control Methods:
  - Linear Quadratic Regulator(LQR)
  - H-Infinity Control
  - PID Control methods
  - State Feedback Control
  - etc.

REM-2011

5

## INTRODUCTION

- Nonlinear Control Methods
  - Feedback Linearization Control Algorithm
  - Backstepping Control
  - Sliding Mode Control
  - Adaptive Fuzzy Control
  - PI-PD-PID like Fuzzy Logic Controllers
  - etc.

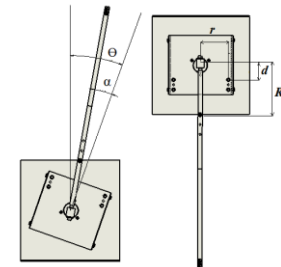
REM-2011

6

## MATHEMATICAL MODELING

### ➤ *Mathematical Model:*

A linear mathematical model for rigid-link flexible joint manipulator can be obtained easily from Lagrange equations. In this Figure,  $\theta$  is rotate angle is deflection angle of end point.



REM-2011

7

## MATHEMATICAL MODELING

- The Lagrangian equation is computed from kinetic and potential energy;

$$L = T - V$$

- The kinetic and potential energy of system can be express as below;

$$T = \frac{1}{2}J_t\dot{\theta}^2 + \frac{1}{2}J_s(\dot{\theta} + \dot{\alpha})^2 \quad V = \frac{1}{2}K_{yay}\alpha^2$$

- The Lagrange equations motion are given as follows;

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \tau_m - B_s\dot{\theta}$$

REM-2011

8

## MATHEMATICAL MODELING

- Solving the Lagrangian equations, equations of motion of system can be obtain as follows ;

$$J_t\ddot{\theta} + J_s(\ddot{\theta} + \ddot{\alpha}) = \tau_m - B_s\dot{\theta} \quad J_s(\ddot{\theta} + \ddot{\alpha}) + K_{yay}\alpha = 0$$

- The relationship between motor torque and applied voltage ;

$$v = iR_m + K_mK_d\omega \quad i = \frac{v}{R_m} - \frac{K_mK_d}{R_m}\omega \quad i = \frac{\tau_m}{K_tK_d}$$

$$\tau_m = \frac{\eta_m\eta_dK_tK_d}{R_m}v - \frac{\eta_m\eta_dK_mK_tK_d^2}{R_m}\dot{\theta}$$

REM-2011

9

## MATHEMATICAL MODELING

- If system states choose as follows;

$$\theta = x_1 \quad \alpha = x_2 \quad \dot{\theta} = x_3 \quad \dot{\alpha} = x_4$$

- System can be defined;

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = \frac{K_{yay}}{J_t}x_2 - \frac{\eta_m\eta_dK_mK_tK_d^2 + B_sR_m}{J_tR_m}x_3 + \frac{\eta_m\eta_dK_tK_d}{J_tR_m}v$$

$$\dot{x}_4 = -\frac{K_{yay}(J_t + J_s)}{J_tJ_s}x_2 + \frac{\eta_m\eta_dK_mK_tK_d^2 + B_sR_m}{J_tR_m}x_3 - \frac{\eta_m\eta_dK_tK_d}{J_tR_m}v$$

REM-2011

10

## MATHEMATICAL MODELING

- So the state space model of the rigid link flexible joint robot can be define as below;

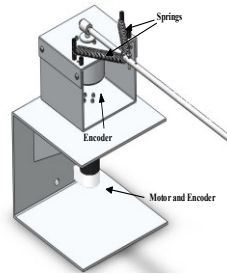
$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{yay}}{J_t} & -\frac{\eta_m\eta_dK_mK_tK_d^2 + B_sR_m}{J_tR_m} & 0 \\ 0 & -\frac{K_{yay}(J_t + J_s)}{J_tJ_s} & \frac{\eta_m\eta_dK_mK_tK_d^2 + B_sR_m}{J_tR_m} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\eta_m\eta_dK_tK_d}{J_tR_m} \\ \frac{\eta_m\eta_dK_tK_d}{J_tR_m} \end{bmatrix} v$$

REM-2011

11

## SYSTEM DESIGN

- To give flexibility of the link two springs attached to the link
- First encoder measure the hub rotation angle
- Second encoder measure the link deflection angle

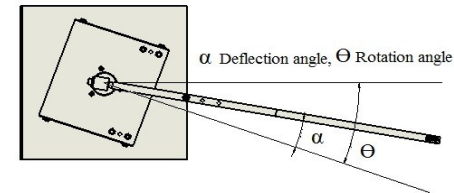


REM- 2011

12

## SYSTEM DESIGN

- The link length of manipulator is 0.4 m
- The spring stiffness rate of flexible joint is 58.86 N/m
- Total mass of the robot is 2.88 kilogram.



REM- 2011

13

## SYSTEM DESIGN

□ Flexible arm has four major parts:

• **Actuator**

- 24V Faulhaber BLDC servomotor (111 mNm stall torque) with a planetary gearbox (246:1 reduction ratio and average 0.3 degree backlash)

• **Incremental encoders**

- Two 512 count/rev. incremental encoders
- Resolution of the link encoder 0.7 degree
- Resolution of the hub encoder 0.0028 degree

REM- 2011

14

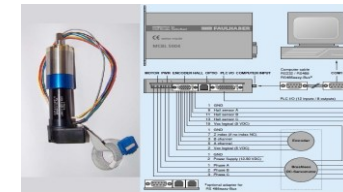
## SYSTEM DESIGN

▪ **BLDC Servomotor driver**

- Faulhaber MCBL 5004 BLDC 50V – 4A servo motor driver

▪ **Controller with its computer interface**

- Controller designed in Simulink® and embedded in ds1103 control board



REM- 2011

15

## SYSTEM DESIGN

Parameters of Flexible Joint Arm

Symbol	Description	Value
$J_{link}$	Inertia of flexible manipulator	0.003882 kgm <sup>2</sup>
$R_m$	Motor resistance	2.1 $\Omega$
$K_g$	Gear ratio of reductor	1/246
$K_m$	Motor constant	0.501 N/(rad/sn)
$K_s$	Flexibility coefficient of joint	58.86 N/m
$M$	Mass of the flexible joint	0.03235 kg
$G$	Gravitational acceleration	-9.81 N/m
$H$	Distance to center of gravity of rotational platform of flexible manipulator	0.06 m
$J_p$	Inertia of rotational platform	0.00075 kgm <sup>2</sup>

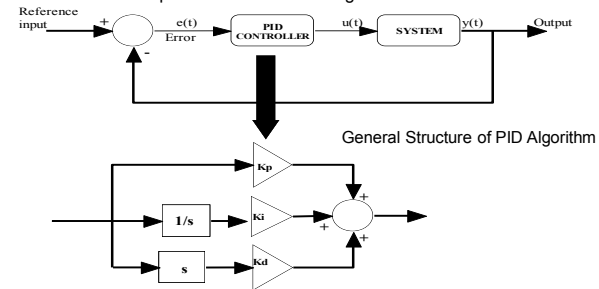


REM- 2011

16

## CONTROLLER DESIGN

► **PID Control:** PID control is one of the classical control methods that is frequently used nowadays especially on industry because of simple structure and working stable.



REM- 2011

17

## CONTROLLER DESIGN

▪ PID algorithm is used to compute the control signal that activates the real system based on the following formula

$$u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{d}{dt} e(t)$$

$$e(t) = r(t) - y(t)$$

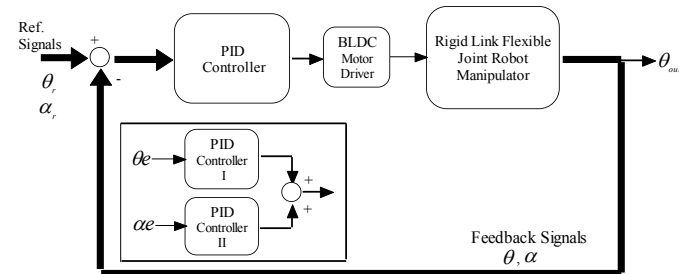
	Rise Time	Overshoot	Settling Time	Steady State Error
$K_p$	Reduce	Increase	Small Change	Reduce
$K_i$	Reduce	Increase	Increase	Eliminate
$K_D$	Small Change	Reduce	Reduce	Small Change

**Effect of PID parameters on system response**

REM- 2011

18

## CONTROLLER DESIGN



The main PID controller scheme of flexible joint robot manipulator

REM- 2011

19

## CONTROLLER DESIGN

- In this controller scheme;
- There exist two different PID controllers to control the rotation angle and end point vibration.
- The error of theta angle is the input signal of PID controller I and the error of alpha angel is the input signal of the PID controller II.
- Control signal of the system is obtained with the addition of the PID controllers outputs.

REM- 2011

20

## CONTROLLER DESIGN

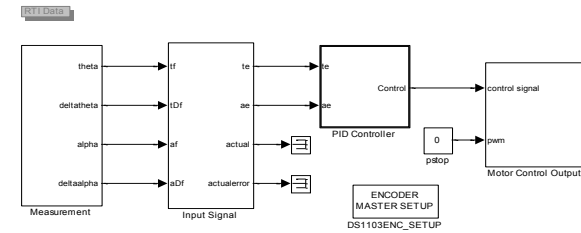
- Duty of measurement block is to measure rotation and deflection angles
- Duty of input block is to obtain the errors between reference and feedback signals.
- Output block produces the control signals and the control block yields the necessary control signals for BLDC servomotor driver.

REM- 2011

22

## CONTROLLER DESIGN

- The controller scheme was designed in Matlab-Simulink and embedded in DS1103 control board

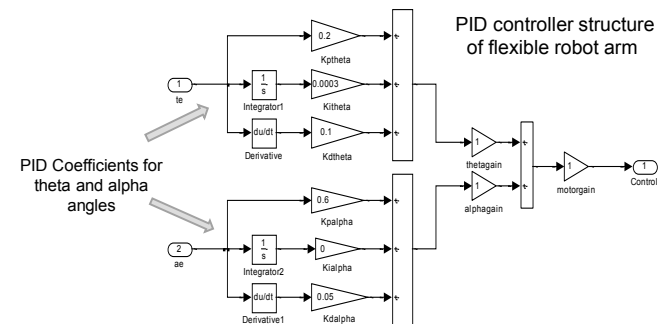


Simulink model of developed PID controller

REM- 2011

21

## CONTROLLER DESIGN



REM- 2011

23

## EXPERIMENTAL RESULTS

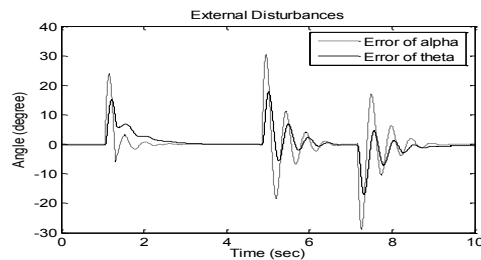
- In order to test performance of the PID controller, several experiments were conducted to the system.
- Experiments can be mainly grouped into four parts: position control, trajectory tracking control and external and internal impulse response experiments.
- To test effectiveness of the each PID parameters, step functions are applied to system

REM- 2011

24

## EXPERIMENTAL RESULTS

- To see the system response in case of the external disturbances, external force impulses were applied to the end-point of the link.



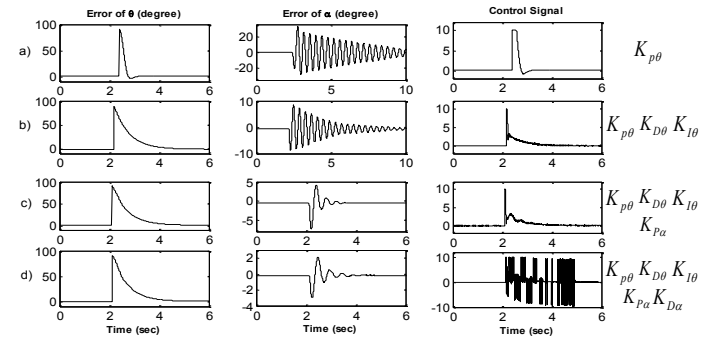
- PID controller eliminates the oscillations in the end effector easily

REM- 2011

26

## EXPERIMENTAL RESULTS

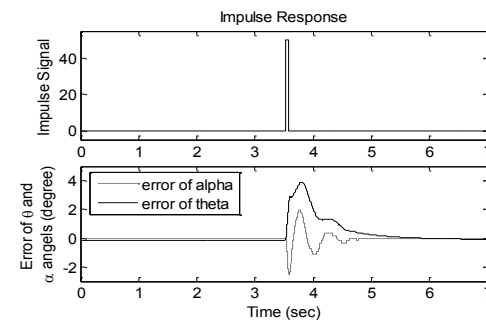
Step responses of flexible joint robot



REM- 2011

25

## EXPERIMENTAL RESULTS

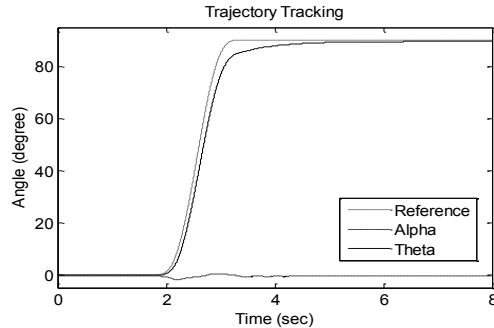


- Developed PID controller is very stable against the impulse signals

REM- 2011

27

## EXPERIMENTAL RESULTS

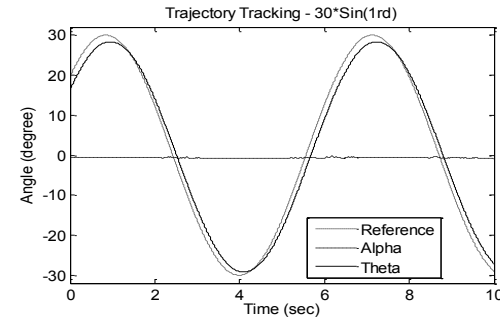


- Maximum 0.1 second phase shift in  $\theta$
- Maximum 0.7 degree vibrations in  $\alpha$

REM- 2011

28

## EXPERIMENTAL RESULTS



- Maximum 0.15 second phase shift in  $\theta$
- 0.35 degree oscillations in  $\alpha$

REM- 2011

29

## EXPERIMENTAL RESULTS

- In the trajectory tracking experiments, two different trajectories were applied to the flexible joint to see tracking performance of the PID.
- In Kane function trajectory tracking experiment, maximum 0.1 second phase shift in  $\theta$  and maximum 0.7 degree vibrations in  $\alpha$  angle are occurred.
- In the sinusoidal trajectory tracking experiment, maximum 0.15 second phase shift in  $\theta$  and 0.35 degree oscillations in  $\alpha$  angle.
- Considering the backlash of planetary gearbox, the results obtained from experiments are satisfactory.

REM- 2011

30

## CONCLUSION

- In this study, position and trajectory tracking control of a rigid-link flexible joint robot arm was implemented with PID controller structure.
- In the position control experiments, no steady-state error and less than 0.7 degree oscillations was achieved in the end-point of the link.
- In the disturbance experiments, PID structure was able to suppress link vibration in a short time (max. 2 sec.).
- Due to the backlash of the planetary gearbox, a small phase shift was occurred in trajectory tracking experiments and a steady state error was occurred in the step response experiments.
- Based on the results of the experiments, it is seen that the control performance of the PID in rigid link flexible joint manipulators is quite good.

REM- 2011

31



***FOR MORE INFORMATION***

**Contact:**

**Ismail H. Akyuz**  
**ismail.akyuz@kocaeli.edu.tr**

**Ali YUKSEL**  
**ali.yuksel3@kocaeli.edu.tr**

**Zafer BINGUL**  
**zaferb@kocaeli.edu.tr**

---

REM-2011